

Drum vortons in high density QCD

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Recently it was shown that high density QCD supports of number of topological defects. In particular, there are $U(1)_Y$ strings that arise due to \bar{K}^0 condensation that occurs when the strange quark mass is relatively large. The unique feature of these strings is that they possess a nonzero K^+ condensate that is trapped on the core. In the following we will show that these strings (with nontrivial core structure) can form closed loops with conserved charge and currents trapped on the string worldsheet. The presence of conserved charges allows these topological defects, called vortons, to carry angular momentum, which makes them classically stable objects. We also give arguments demonstrating that vortons carry angular momentum very efficiently (in terms of energy per unit angular momentum) such that they might be the important degrees of freedom in the cores of neutron stars.

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I. INTRODUCTION

In the high baryon density regime it is well established that QCD behaves like a BCS superconductor where the condensate of electron Cooper pairs is replaced by a diquark condensate which has a very specific structure [1, 2, 3, 4, 5] (see [6] for a review of high density QCD). The presence of a condensate of quark Cooper pairs breaks certain symmetries which are respected at zero baryon density. Recently, it has been shown that a consequence of this symmetry breaking is the appearance of various topological defects [7, 8, 9, 10, 11, 12]. This leads to the exciting possibility that such objects (which do not exist in the Standard Model in the vacuum) may be present within the interior of compact stars where such large densities may be realized [13].

In addition to a diquark condensate, it may be energetically favorable for a K^0 condensate to form for the physical value of the strange quark mass, $m_s \simeq 150$ MeV $\gg m_u, m_d$ [14, 15, 16, 17]. The hypercharge $U(1)_Y$ symmetry is spontaneously broken in this case, leading to the formation of global $U(1)_Y$ strings [9]. The main feature which distinguishes these strings from other types of global strings present at high density, namely $U(1)_B$ and $U(1)_A$ strings [9], is the presence of a nonzero K^+ condensate which forms on the core of the string [10, 12]. This means that the electromagnetic $U(1)_{EM}$ symmetry is spontaneously broken in the core of the string, and we will refer to them as superconducting K^+ -strings. Analytical calculations demonstrated explicitly that K^+ condensation only occurs for a relatively large strength of the kaon condensate, parameterized by the angle θ_{K^0} (see below). If $\theta < \theta_{\text{crit}} \sim \sqrt{(m_d - m_u)/m_s}$, the symmetry is restored in the core and K^+ condensation does not occur [12]. We should note that this phenomenon where the symmetry is not restored in the core of the vortex was pointed out

long ago by Witten [18] and has received a lot of attention since in the context of cosmology and cosmic strings [19, 20, 21, 22]. It is interesting to note that an example of this phenomenon has been found in the context of the $SO(5)$ theory of high temperature superconductivity [23], where the conventional vortices in Type II superconductors might have antiferromagnetic cores. Furthermore, there has been recent experimental evidence supporting the presence of the antiferromagnetic condensate in superconducting vortices [24]. Therefore, the study of vortices when the symmetry is not restored in the core is not a purely academic question.

In addition to the work of Witten [18], the subsequent studies demonstrated that the presence of a condensate localized on the string may lead to the classical stability of superconducting string loops [19, 20]. The first class of superconducting string loops, called “springs”, were characterized by a nonzero spacelike current trapped on the string worldsheet [19, 20]. It is well known that a loop of a normal global string with no condensate in the core is unstable in the vacuum. Upon formation the loop will shrink and eventually disappear through the emission of particles. The hope was that if there is a persistent superconducting current trapped on the string, then this current could in turn balance the string tension and prevent the loop from shrinking. Such a configuration would have a total energy $E \sim \mu_{\text{string}} L + J^2 L$, where μ_{string} is the string tension (or energy per unit length), $J \sim 1/L$ is the current, and L is the length of the loop. Since the energy in the global string is linear in L and the energy due to the current goes like $1/L$, there should exist a classically stable configuration for nonzero length L_{min} . However, it was realized that springs do not exist in a large region of parameter space due to the fact that as the loop shrinks the current becomes larger, which quenches the value of the condensate on the string [20, 22]. Physically, the current acts as a positive mass squared in the Lagrangian, which decreases the size of the con-

densate in the core. In most cases the maximum current which can occur before the condensate is quenched is less than the current which is needed to stabilize the string loop. Therefore, though such string may become stable configuration (due to the special fine tuning), it is not a typical, but rather exceptional case.

A more general type of topological defect was introduced by Davis and Shellard in [21, 25, 26] which has two types of conserved charges trapped on the string worldsheet. The first is a topological charge which had been included the analysis of springs, and the second is a Noether charge which is also trapped on the string worldsheet. The difference with the previous case is that a more general solution is “stationary” but not “static”; rather it has an explicit time-dependent phase $e^{i\omega t}$. This factor leads to a conserved charge trapped on the core, and the configuration can be stabilized due to the conservation of the corresponding charge. The time-dependent configuration becomes the lowest energy state in the sector with a given nonzero charge. A similar idea was advocated by Coleman [27] in his construction of Q-balls, stable objects with a time-dependent wave function. This class of objects generally possess nonzero angular momentum and charge, which lead these quasiparticles to be referred to as vortons. In this paper we will apply these ideas to K^0 -strings with K^+ -cores [12] to construct the vortons in the CFL+ K^0 phase of high density QCD. One difference between our case and other theories where vortons are present is that our vortons have a domain wall [8] which stretches across the surface of the vorton like a soap bubble. This particular type of vorton with a domain wall attached was discussed recently within the linear sigma model at nonzero temperature [28]. We should note that we will not be addressing the issue of vorton formation in this paper. Rather, we will assume that there is some nonzero probability for the formation of such topological defects when the CFL+ K^0 phase occurs. We refer the reader to [29] for a detailed study on vorton formation. Recently, vortons have also been constructed in the context of the Zhang’s $SO(5)$ theory of high temperature superconductivity [30]. This analogy between astrophysics/cosmology and condensed matter physics provides a unique opportunity to study the cosmological/astrophysical phenomena by doing laboratory experiments in condensed matter physics. Over the last few years several experiments have been done to test ideas drawn from cosmology (see the review papers [31, 32] for further details).

This paper is organized in the following manner. In section II we will briefly review the properties of the CFL+ K^0 phase of high density QCD as well as the properties of the superconducting K -string as presented in [12]. In section III, we will show that classically stable loops of superconducting K -strings can exist. The domain walls which are attached to such vorton configurations will be discussed in section IV. Section IV will also contain numerical estimates of various properties of the vortons such as their size and the Magnus force that

leads to further stability. Finally, we end this paper in section V with concluding remarks where we argue that the vortons can carry the angular momentum very efficiently; they need the least energy per given angular momentum.

II. THE CFL+ K^0 PHASE OF HIGH DENSITY QCD AND SUPERCONDUCTING K -STRINGS

The ground state of the high density phase of QCD is characterized by a diquark condensate [2, 3, 4, 5] analogous to the condensate of electron Cooper pairs present in a conventional superconductor. This phase of QCD is referred to as a color superconducting phase and the form of the condensate for $N_c = N_f = 3$ (CFL phase) has a very specific structure given by [4]:

$$\begin{aligned}\langle q_{L\alpha}^{ia} q_{L\beta}^{jb} \rangle^* &\sim \epsilon_{\alpha\beta\gamma} \epsilon^{ij} \epsilon^{abc} X_c^\gamma, \\ \langle q_{R\alpha}^{ia} q_{R\beta}^{jb} \rangle^* &\sim \epsilon_{\alpha\beta\gamma} \epsilon^{ij} \epsilon^{abc} Y_c^\gamma,\end{aligned}\quad (1)$$

where L and R represent left and right handed quarks, α , β , and γ are the flavor indices, i and j are spinor indices, a , b , and c are color indices, and X_c^γ and Y_c^γ are complex color-flavor matrices describing the Goldstone bosons. In order to describe the light degrees of freedom in a gauge invariant way, one introduces the color singlet field Σ

$$\Sigma_\gamma^\beta = XY^\dagger = \sum_c X_c^\beta Y_\gamma^{c*}, \quad (2)$$

One can describe the Goldstone bosons contained in the field Σ using the following effective Lagrangian [14, 15, 16, 17]:

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \frac{f_\pi^2}{4} \text{Tr} [\nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2 \partial_i \Sigma \partial_i \Sigma^\dagger], \\ &+ 2A [\det(M) \text{Tr}(M^{-1} \Sigma + h.c.)], \\ \nabla_0 \Sigma &= \partial_0 \Sigma + i \left(\frac{MM^\dagger}{2p_f} \right) \Sigma - i \Sigma \left(\frac{M^\dagger M}{2p_f} \right),\end{aligned}\quad (3)$$

where the matrix $\Sigma = \exp(i\pi^a \lambda^a / f_\pi)$ describes the octet of Goldstone bosons with the $SU(3)$ generators λ^a normalized as $\text{Tr}(\lambda^a \lambda^b) = 2\delta^{ab}$. The quark mass matrix in Eq.(3) is given by $M = \text{diag}(m_u, m_d, m_s)$. The constants f_π , v and A have been calculated in the leading perturbative approximation and are given by [33, 34]:

$$f_\pi^2 = \frac{21 - 8 \ln 2}{18} \frac{\mu^2}{2\pi^2}, \quad v^2 = \frac{1}{3}, \quad A = \frac{3\Delta^2}{4\pi^2}. \quad (4)$$

In [14, 15, 16, 17] it was noticed that for a physical value of the strange quark mass, $m_s > 60$ MeV, K^0 condensation would occur and that $\Sigma_o = \text{diag}(1, 1, 1)$ would no longer represent the true ground state of the CFL phase [15]. Instead, Σ_o would be rotated in some different direction in flavor space. The instability of the ground state originates from the addition of the covariant derivative

in (3). In the following we will consider the physical case where isospin symmetry is not exact ($m_d > m_u$) and overall electric charge neutrality, such that K^0 condensation occurs. The appropriate expression for Σ_o describing the K^0 condensed ground state can be parameterized as:

$$\Sigma_o = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{K^0} & \sin \theta_{K^0} e^{-i\varphi} \\ 0 & -\sin \theta_{K^0} e^{i\varphi} & \cos \theta_{K^0} \end{pmatrix}, \quad (5)$$

where φ describes the corresponding Goldstone mode and θ_{K^0} describes the strength of the kaon condensation with [15]:

$$\cos \theta_{K^0} = \frac{m_0^2}{\mu_{\text{eff}}^2}, \quad a = \frac{4A}{f_\pi^2} = \frac{3\Delta^2}{\pi^2 f_\pi^2}, \quad (6)$$

$$m_0^2 \equiv a m_u (m_d + m_s), \quad \mu_{\text{eff}} \equiv \frac{m_s^2}{2p_F}.$$

In order for kaon condensation ($\theta_{K^0} \neq 0$) to occur, we must have $m_0 < \mu_{\text{eff}}$. This leads to the breaking of the hypercharge $U(1)_Y$ symmetry. As discussed in [15], the lightest degrees of freedom in the CFL+ K^0 phase are the K^0 and K^+ mesons.

Before we proceed, let us make the following remark regarding the description of Goldstone particles and topological defects based on the effective Lagrangian approach. To formulate the problem in a more specific way, let us remind the reader that, in general, the effective Lagrangian describing the Goldstone modes can be represented in many different forms as long as the symmetry

properties are respected. The results for the amplitudes describing the interaction of the Goldstone particles do not depend on a specific representation used. A well-known example of this fact is the possibility of describing the π meson properties by using a linear σ model as well as a non-linear σ model (and many other models which satisfy the relevant symmetry breaking pattern). The results remain the same if one discusses the local properties of the theory (such as the $\pi\pi$ scattering length) when the π meson is considered as a small quantum fluctuation rather than a large background field. It may not be the case when π represents a large background field in which case some numerical difference between different representations of the effective Lagrangian may occur. Roughly speaking, the source of the difference is an inequality $\pi(x) \neq \sin \pi(x)$ for large global background fields such as a string solution which is the subject of this letter. Therefore, in what follows we assume that the qualitative effects which follow from the low-energy Lagrangian remain untouched when a different representation for the fields/interactions is used as long as the symmetry properties are respected. Our assumption is based on the experience [12] when two different representations for K fields lead to the similar numerical results.

With this remark in mind, we shall use the effective Lagrangian by expanding the full effective Lagrangian given by (3) to fourth order in the fields K^0, K^+ . We expect that this Lagrangian captures the essential physics of the K^0 and K^+ mesons because it respects all relevant symmetries:

$$\mathcal{L} = \frac{1}{6f_\pi^2} ((\bar{K}^0 \overleftrightarrow{D}_\mu K^0)^2 + (\bar{K}^+ \overleftrightarrow{D}_\mu K^+)^2 + (\bar{K}^+ \overleftrightarrow{D}_\mu K^0)(\bar{K}^0 \overleftrightarrow{D}_\mu K^+) + (\bar{K}^+ \overleftrightarrow{D}_\mu K^+)(\bar{K}^0 \overleftrightarrow{D}_\mu K^0))$$

$$+ |D_\mu K^0|^2 + |D_\mu K^+|^2 - m_0^2 |K^0|^2 - m_+^2 |K^+|^2 + \frac{m_0^2}{6f_\pi^2} |K^0|^4 + \frac{m_+^2}{6f_\pi^2} |K^+|^4 + \frac{(m_0^2 + m_+^2)}{6f_\pi^2} |K^0|^2 |K^+|^2 \quad (7)$$

where the covariant derivative is defined by:

$$D_0 \equiv (\partial_0 - i\mu_{\text{eff}}), \quad D_i \equiv v\partial_i,$$

$$\bar{\phi}_1 \overleftrightarrow{D}_\mu \phi_2 \equiv \bar{\phi}_1 (D_\mu \phi_2) - (\overline{D_\mu \phi_1}) \phi_2, \quad (8)$$

$$(\overline{D_0 \phi_1}) \equiv (\partial_0 + i\mu_{\text{eff}}) \bar{\phi}_1, \quad (\overline{D_i \phi_1}) \equiv v\partial_i \bar{\phi}_1$$

and the masses are given by:

$$m_0^2 = a m_u (m_s + m_d), \quad m_+^2 = a m_d (m_s + m_u) \quad (9)$$

It is necessary to keep the fourth order terms involving derivatives such as $\partial_0 K^+, \partial_i K^+$ in (7) for a discussion of vortons (the reason for this will become apparent in the next section). The effective Lagrangian given in (7) will be used throughout the rest of this paper.

If one neglects the fourth order terms with derivatives $\sim \mu_{\text{eff}} \neq 0$ only, the Lagrangian (7) reduces to one discussed in [12] expressed in terms of the single complex doublet $\Phi = (K^+, K^0)$:

$$\mathcal{L} = |\partial_0 \Phi|^2 - v^2 |\partial_i \Phi|^2$$

$$- \lambda \left(|\Phi|^2 - \frac{\eta^2}{2} \right)^2 - \delta m^2 \Phi^\dagger \tau_3 \Phi, \quad (10)$$

where τ_3 is the third Pauli matrix and the various parameters are given by:

$$\lambda \simeq \frac{4\mu_{\text{eff}}^2 - m_0^2}{6f_\pi^2}, \quad \lambda\eta^2 = \mu_{\text{eff}}^2 - \frac{m_0^2 + m_+^2}{2},$$

$$\delta m^2 = \frac{m_+^2 - m_0^2}{2} = \frac{a}{2} m_s (m_d - m_u) \quad (11)$$

As we know, the Lagrangian (10) admits superconducting string solutions, which is not immediately obvious when \mathcal{L} is written in the representation (7). In everything that follows we will take $m_u \neq m_d \ll m_s$ so that the $SU(2)$ isospin symmetry is broken. Given this, the effective Lagrangian (7) is invariant under the $U(1)_Y \times U(1)_{EM}$ symmetry group, which correspond to independent phase rotations for the K^0 and K^+ Goldstone bosons. When the explicit symmetry breaking terms are zero, the symmetry becomes $SU(2) \times U(1)$.

We will briefly review the basic characteristics of the superconducting K strings as presented in [12]. If we consider the case where $\mu_{\text{eff}} > m_0$, then the K^0 field in (7) acquires a negative mass squared and this signals the formation of a nonzero K^0 condensate which spontaneously breaks the $U(1)_Y$ symmetry. The breaking of this $U(1)_Y$ symmetry leads to the existence of classically stable nontrivial solutions to the time independent equations of motion (strings). This particular type of topological defect is characterized by the variation of the phase of the field K^0 from 0 to 2π around a point where the vacuum expectation value of K^0 vanishes. Outside of this region where the field approaches its vacuum expectation value over a distance scale $\sim 1/m$, where m is the mass scale associated with K^0 . In addition to the formation of K^0 strings, in a certain range of parameter space it is energetically favorable for K^+ condensation to occur at the center of the string as discussed in [12], which leads to strings that are superconducting. The superconducting strings can be described using the following time independent ansatz:

$$K_{\text{string}}^0(r, \phi) = \frac{\eta'}{\sqrt{2}} f(r) e^{i\phi}, \quad (12)$$

$$K_{\text{cond}}^+(r) = \frac{\sigma}{\sqrt{2}} g(r), \quad (13)$$

where $\eta'/\sqrt{2} = \sqrt{(\mu_{\text{eff}}^2 - m_0^2)/(2\lambda_0)}$ is different from the vacuum expectation value for the field $\langle \Phi \rangle = \eta/\sqrt{2}$ (see (10)) by the size of the symmetry breaking term $\sim \delta m^2$, ϕ is the azimuthal angle in cylindrical coordinates, $\sigma/\sqrt{2}$ is the value of the condensate on the string core, and $f(r)$ and $g(r)$ are solutions to the equations of motion which obey the boundary conditions $f(0) = 0$, $f(\infty) = 1$ and $g'(0) = 0$, $g(\infty) = 0$. This configuration is the one described above where the field K^0 vanishes at the center of the string and goes to its vacuum expectation value at ∞ , with a nonzero K^+ condensate that exists only on the string core. The functions $f(r)$ and $g(r)$ can be approximated by the following functions which obey the appropriate boundary conditions:

$$\begin{aligned} f(r) &\approx (1 - e^{-\beta r}), \\ g(r) &\approx e^{-\kappa r} (1 + \kappa r), \end{aligned} \quad (14)$$

where $\beta \simeq \sqrt{\mu_{\text{eff}}^2 - m_0^2}$ and $\kappa \simeq \delta m$ are the approximate inverse widths of the string core and condensate respectively. In addition, the value of the condensate at the

center of the string can be estimated by substituting the approximate solutions (12) and (14) into the Hamiltonian and minimizing the energy with respect to the parameter σ .

III. K -VORTONS

Now that we have reviewed the basic ideas behind superconducting K strings, we will proceed to show that superconducting string loops can exist as classically stable objects which are supported by the presence of two conserved charges which become trapped on the string worldsheet, called vortons. These quasiparticles have been widely discussed in the context of cosmology [18, 19, 20, 21, 22, 25, 26, 28]. In our case, high density QCD, we have the benefit of having an effective Lagrangian that contains parameters that have already been calculated.

A. Springs vs. Vortons

Shortly after the pioneering paper of Witten [18] on superconducting strings, there was a lot of interest in the idea that superconducting strings loops could be supported by the presence of persistent currents [19, 20, 22]. We will consider a large loop of string of radius $R \gg \delta$, where δ is the string thickness, so that curvature effects can be neglected. The z -axis is defined along the length of the string, varying from 0 to $L = 2\pi R$ as one goes around the loop. The superconducting current can be described by including a phase in the K^+ field, $K^+ \rightarrow K^+ e^{ikz}$. Following [18], we define a charge N which is topologically conserved:

$$N = \oint_C \frac{dz}{2\pi} \left(\frac{d\alpha}{dz} \right) = kR, \quad (15)$$

where the path C is defined along the string loop and $\alpha = kz$ is the phase. Since the field must be single valued as one goes around the loop, k can be interpreted as a winding number density $k = N/R$, with N constrained to be an integer. If N is an integer then it cannot change continuously. This means that there is a persistent current associated with the conserved quantity N . The only way that N can unwind is through a tunneling process whereby the condensate is quenched to down to zero on the string, allowing the winding number to decrease from N to $N - 1$ [35].

The energy of this configuration has the form:

$$\begin{aligned} E &= \mu_{\text{string}} L + v^2 k^2 L \Sigma \\ &= \mu_{\text{string}} L + (2\pi)^2 v^2 \frac{N^2}{L} \Sigma \end{aligned} \quad (16)$$

where μ_{string} is the K^0 string tension, winding number density k is expressed in terms of the conserved charge

N , (15), and the quantity Σ is defined as the integral of $|K^+|^2$ over the string cross section ¹

$$\Sigma \equiv \int_{\times} d^2r |K^+|^2. \quad (17)$$

This energy has a non-trivial minimum with respect to the loop length L , thus it was originally believed that springs are stable semi-topological objects.

However, later on it was realized that the spring cannot carry arbitrarily large currents or winding number densities $k = N/R$. The addition of the z -dependent phase contributes an effective positive mass-squared term to the Lagrangian:

$$\delta\mathcal{L} = -v^2 |\partial_z K^+|^2 = -v^2 k^2 |K^+|^2. \quad (18)$$

As the loop with a conserved and nonzero charge N (defined at the moment of loop formation) shrinks to reach the energetically favorable length, k increases, the effective mass squared of K^+ on the string core also increases hence decreasing the strength of the condensate inside the core. Eventually it may be no longer energetically favorable for K^+ to condense inside the string core, and superconductivity on the string will be destroyed with $|K^+|$ quenched down to 0 on the string. In most models discussed in the context of cosmology [20, 22], quenching occurs before the spring reaches its equilibrium length and hence no stable configurations exist.

However, this is not the end of the story. As Davis and Shellard originally pointed out there exists a more general type of topological defect which is stabilized by angular momentum [21, 25, 26]. These types of objects are referred to as vortons and have been widely discussed in the context of cosmology and cosmic strings (see [36] for a review and [37] for recent work). As well as the topological charge present in the spring configurations discussed previously, vortons also have a conserved Noether charge on the string core. The amount of charge present on the string is proportional to the parameter ω in the time dependent phase $K^+ \rightarrow K^+ e^{-i\omega t}$. The addition of this phase leads to a nonzero Noether charge given by:

$$Q = \int d^3r j^0 = \omega L \Sigma. \quad (19)$$

The addition of time dependent phase also contributes to the effective mass squared of the K^+ field as in (18), only having the opposite sign:

$$\delta\mathcal{L} = |\partial_0 K^+|^2 = +\omega^2 |K^+|^2. \quad (20)$$

Yet ω enters the energy with the same sign as k :

$$\begin{aligned} E &= \mu_{\text{string}} L + (v^2 k^2 + \omega^2) \Sigma L \\ &= \mu_{\text{string}} L + (2\pi)^2 v^2 \frac{N^2}{L} \Sigma + \frac{Q^2}{\Sigma L}, \end{aligned} \quad (21)$$

¹ The quantity Σ should not be confused with the matrix Σ_γ^β in the last section that describes the octet of Goldstone bosons.

and therefore the energy still has a nontrivial minimum with respect to the loop length.

Thus, from (20) we see that a nonzero ω will counteract the quenching effect of k , increasing the value of the condensate on the string (anti-quenching) as the string loop shrinks [25]. As discussed in [25], when the loop shrinks $\omega/(vk)$ tends to 1, meaning that at equilibrium length the quenching and anti-quenching effects approximately cancel each other out, leaving a stable vorton behind.

Note that the results discussed above do not rely on whether the condensate on the string is electrically charged. As Davis and Shellard originally pointed out [25], the stability of the vortons is purely mechanical and not electromagnetic in origin. The reason is simple, a vorton with nonzero N and Q has nonzero angular momentum. The fact that the vorton is spinning and angular momentum is conserved leads to the classical stability of these objects. Therefore, the addition of electromagnetic effects should not change the qualitative behavior that will be discussed below, and therefore we neglect the electromagnetic contribution in the present work.

B. Vortons in the CFL+ K^0 phase of high density QCD

In order to describe a vorton in our case, we will add a time and z dependent phase in the standard form to the string-condensate solution (12) presented in the previous section:

$$\begin{aligned} K^0 &= K_{\text{string}}^0(r, \phi), \\ K^+ &= K_{\text{cond}}^+(r) e^{-i\omega t + ikz}. \end{aligned} \quad (22)$$

Recall that z is defined as the coordinate which runs along the length of the string and varies from 0 to $L = 2\pi R$ as one goes around a loop of radius R . The loop is assumed to be large, $R \gg \delta$ (δ is the typical string thickness), so that we can neglect curvature effects and consider a straight string.

We can substitute these expression into the original Lagrangian (7) and obtain the Lagrangian describing the dynamics in the two transverse dimensions:

$$\begin{aligned} \mathcal{L} &= -v^2 |\partial_i K^0|^2 - v^2 |\partial_i K^+|^2 + M_0^2 |K^0|^2 + M_+^2 |K^+|^2 \\ &\quad - \lambda_0 |K^0|^4 - \lambda_+ |K^+|^4 - \zeta |K^0|^2 |K^+|^2 \end{aligned} \quad (23)$$

where i runs over x, y and

$$\tilde{\omega} = \omega + \mu_{\text{eff}} \quad (24)$$

is the effective frequency of K^+ field, and the parameters of (23) are given by:

$$\begin{aligned} M_0^2 &= \mu_{\text{eff}}^2 - m_0^2, & M_+^2 &= \tilde{\omega}^2 - v^2 k^2 - m_+^2 \\ \lambda_0 &= \frac{4\mu_{\text{eff}}^2 - m_0^2}{6f_\pi^2}, & \lambda_+ &= \frac{4(\tilde{\omega}^2 - v^2 k^2) - m_+^2}{6f_\pi^2}, \\ \zeta &= \frac{(\tilde{\omega} + \mu_{\text{eff}})^2 + 4\tilde{\omega}\mu_{\text{eff}} - v^2 n^2 - m_+^2 - m_0^2}{6f_\pi^2} \end{aligned}$$

In simplifying (7) to (23) we have ignored all fourth order terms in fields which have derivatives in x, y directions since these variations change the profile (as a function of r) of the string itself, but do not influence the effects that are the main subject of this work – the formation of a closed loop of the string, a vorton.

It is important to note that fourth order couplings in (23) are strongly dependent on ω and k . This property is what distinguishes our model from the models considered in the context of cosmology [22]. However, the main feature of the time-dependent ansatz (22) which leads to the existence of stable vortons does not depend on these small differences; it remains the same as discussed earlier in different models in the cosmological context [22]. This is because the stability of vortons is not related to the specific properties of the Lagrangian, but rather it is guaranteed by the conservation of topological charge (15) and the Noether charge:

$$\begin{aligned} Q &= \int d^3r j^0 \\ &= \frac{L}{2} \int d^2r |K^+|^2 [2(\omega + \mu_{\text{eff}})] \\ &\simeq L\tilde{\omega}\Sigma. \end{aligned} \quad (25)$$

which reduces to the expression (19) with the replacement $\omega \rightarrow \tilde{\omega} = \omega + \mu_{\text{eff}}$. In the above expression we have omitted higher order terms² in derivatives and/or fields to simplify the expression for the charge Q (25). The presence of time dependence leads to a configuration with non-zero conserved Noether charge. Such an object, which is stable due to the conservation of a Noether charge, is called a Q-ball [27]. Vortons have been referred to as semi-topological defects [21] due to the fact that they are partially stabilized by topology and partially stabilized by the presence of a conserved Noether charge, similar to Q-balls.

Thus K^+/K^0 string loops in our model are always charged and this charge has to be taken into account when studying their dynamics. In particular, even if the K^+ field originally has no explicit time dependence, i.e. $\omega = 0$ and $\tilde{\omega} = \mu_{\text{eff}}$, explicit time dependence will appear in the process of loop shrinking to preserve the Noether charge that was present at the moment of formation.

Now we proceed to study what values $\tilde{\omega}$ and k can assume. These values are clearly not arbitrary since the masses and the couplings in the Lagrangian (23) depend on $\tilde{\omega}$ and k . Thus, for the vorton to exist, $\tilde{\omega}$ and k must not destroy the K^+ condensate of the superconducting K^0/K^+ string. The constraints on the parameters in (23)

which guarantee superconductivity have been discussed in detail by [20] and can be stated as follows:

1. It must be energetically favorable for K^0 to condense in the vacuum, i.e. in vacuum $\langle K^0 \rangle \neq 0$. This guarantees that the $U(1)_Y$ symmetry is spontaneously broken and a K^0 string can form.
2. It must be energetically unfavorable for K^+ to condense in the vacuum, i.e. in vacuum $\langle K^+ \rangle = 0$. This guarantees that K^+ does not condense outside the string and is bound to the string core. This constraint requires that the effective mass squared of K^+ must be positive off of the string core.
3. It must be energetically favorable for K^+ condensation to occur on the string core. A necessary condition for this is that the effective mass squared of K^+ must be negative inside the string core.
4. A sufficient condition for 3. is that total energy associated with K^+ condensation inside the string core must be negative.

The first 3 constraints can be summarized in terms of the parameters (25) as:

$$\begin{aligned} 1. & \quad \frac{M_0^4}{\lambda_0} > \frac{M_+^4}{\lambda_+} \\ 2. & \quad \frac{\zeta}{2\lambda_0} M_0^2 > M_+^2 \\ 3. & \quad M_+^2 > 0 \end{aligned} \quad (26)$$

Before we make numerical estimates for the parameters ω, k , notice that the approximate degeneracy between K^0 and K^+ implies that $M_+^2 \simeq M_0^2$ and therefore

$$\tilde{\omega}^2 - v^2 k^2 - m_+^2 \approx \mu_{\text{eff}}^2 - m_0^2, \quad (27)$$

which suggests that the typical scale for ω and k is the scale $\mu_{\text{eff}} \sim m_s^2/(2\mu) \sim 30$ MeV. Our variational calculations (similar to the one performed in [12] using the ansatz (14)) used to obtain the allowed values of ω and k support this estimate. When values of $\tilde{\omega}$ and k satisfying all 4 conditions are plotted we see that superconducting strings exist for $\tilde{\omega}$ from 25 to 45 MeV.

The upper limit on $\tilde{\omega}$ has to do with the fact that for large values of $\tilde{\omega}$ the parameters M_+^2 and ζ break the degeneracy between the K^0 and K^+ fields. Moreover, the increase of M_+^2 and ζ with $\tilde{\omega}$ cannot be cancelled out simultaneously by an increase in k . Thus for large $\tilde{\omega}$, ζ becomes too large and the energy associated with a K^+ condensate in the core is no longer negative.

The total energy of this field configuration can also be computed from (7). The energy is given as usual by the integral of the T^{00} component of the energy-momentum tensor:

$$\begin{aligned} E &= L \int d^2r (v^2 |\partial_i K^0|^2 + v^2 |\partial_i K^+|^2 - M_0^2 |K^0|^2 \\ &\quad - M_+^2 |K^+|^2 + \lambda_0 |K^0|^4 + \lambda'_+ |K^+|^4 + \zeta' |K^0|^2 |K^+|^2) \end{aligned} \quad (28)$$

² As we have already mentioned these higher order terms in the effective description lead, in general, to some difference in the definition of the fields. In particular, we could define the fundamental field K^+ as the phase of Σ_γ^β (2) or we could define K^+ as $\sin(K^+)$ etc. We do not expect that this ambiguity can change the qualitative results which follow.

where the coefficients are given by:

$$\begin{aligned} M_0^2 &= \mu_{\text{eff}}^2 - m_0^2, \quad \lambda_0 = \frac{4\mu_{\text{eff}}^2 - m_0^2}{6f_\pi^2}, \\ M_+^{\prime 2} &= \mu_{\text{eff}}^2 - \omega^2 - v^2 k^2 - m_+^2, \\ \lambda_+^{\prime} &= \frac{4(\mu_{\text{eff}}^2 - \omega^2 - v^2 k^2) - m_+^2}{6f_\pi^2}, \\ \zeta' &= \frac{8\mu_{\text{eff}}^2 - \omega^2 - v^2 k^2 - m_+^2 - m_0^2}{6f_\pi^2} \end{aligned} \quad (29)$$

In the case when $\omega = k = 0$ one reproduces the energy for the string obtained from eq. (10). The part of the energy (28) associated with a single K^0 vortex of length L without a condensate in the center (terms involving only K^0 in (28)) is given to logarithmic accuracy as:

$$E_{K^0} = 2\pi R (\pi \eta'^2 v^2 \ln(\beta\Lambda)), \quad \eta'^2 = \frac{\mu_{\text{eff}}^2 - m_0^2}{\lambda_0} \quad (30)$$

where β is the inverse width of the string's core introduced in the ansatz (14) and Λ is a long distance cutoff which is introduced in order to control the logarithmic divergence which appears due to the large distance variation of the phase. The cutoff is typically the distance between strings or the radius of curvature, and since we will be considering loops of strings in this paper, the natural correspondence to make is $\Lambda \simeq R$. The additional energy of the K^+ condensate in the core of the string (due to the nonzero values of ω, k) has the leading term behavior:

$$\begin{aligned} E_{K^+} &\simeq L(\omega^2 + v^2 k^2) \Sigma \\ &\simeq \frac{Q^2}{2\pi R \Sigma} + \frac{2\pi v^2 N^2 \Sigma}{R}, \end{aligned} \quad (31)$$

where we expressed ω, k in terms of the conserved charges Q, N . Note this is only an approximate expression for the energy of the condensate and that we have neglected various higher order terms in K^0, K^+ in eq. (28) as well as in the definition of the charge Q (25). As we mentioned earlier, these higher order terms reflect the ambiguity in the description of solitons using the effective Lagrangian approach when $K^0/f_\pi \sim 1$. These terms effectively play a role by determining the magnitude of the K^+ condensate in the core represented by the parameter Σ in our calculations. Once the presence of a K^+ condensate is established, these terms can change some numerical results, but we do not expect that these terms can change our qualitative results because the existence of vortons is based on conservation of charges rather than on the specific properties of the field representations used in this paper. In other words, once the parameters are such that a K^+ condensate forms in the core of the vortex, the vorton can also form. We use the simplest possible expressions for the relevant parameters in order to illuminate the fact that stability occurs for a nonzero value of R . Thus, as discussed before, the total energy $E = E_{K^0} + E_{K^+}$ has

a minimum with respect to R at which a stable vorton exists. With our parameters it happens at R_0 given by,

$$(2\pi R_0)^2 \simeq \frac{Q^2 + (2\pi)^2 v^2 N^2 \Sigma^2}{\pi \Sigma \eta'^2 v^2 \ln(\beta\Lambda)}. \quad (32)$$

At this point in our discussion the size of vortons is not constrained in any way; it could be arbitrarily large, similar to the cosmic string vortons. However, when an explicit symmetry breaking term is taken into account, the vorton size can not be arbitrarily large. Rather, the size will be constrained by the strength of an additional force due to the domain wall attached to the string (see the next section).

One should emphasize at this point that the source of this stability is purely mechanical, and not related to the electromagnetic interactions. This is in contrast with the suggestion made in [10], where it was mentioned that it may be possible to have classically stable K -vortons in high density QCD due to a persistent superconducting current trapped on the string. As we have demonstrated above, the source of the vorton stability has a quite different origin. We expect that the maximum electromagnetic current which can occur in the system (before the K^+ condensate is quenched) is less than the current which is required to stabilize the string loop, as it was demonstrated to happen in most cases [20] and [22] where a similar problem was previously analyzed.

The stability of the vortons can also be demonstrated explicitly in a different way. As Davis and Shellard originally pointed out, the source of this stability is purely mechanical. The presence of time dependence in (22) allows the vortons to spin and carry angular momentum. The conservation of angular momentum is reflected by the conservation of the topological and Noether charges Q and N , respectively. We can easily calculate the approximate angular momentum carried by a vorton from the energy-momentum tensor obtained from (7)

$$M^{ij} = \int d^3r (x^i T^{0j} - x^j T^{0i}). \quad (33)$$

The angular momentum carried by a single vorton (22) is approximately:

$$M \simeq 2\pi R^2 k \tilde{w} \Sigma. \quad (34)$$

The direction of M is perpendicular to the surface formed by the vorton. From the expression (34) for the angular momentum, we can see that that $M \simeq N \cdot Q$ is proportional to the classically conserved quantities N, Q .

Although our K^+ field is electrically charged, we have not mentioned or included interactions with electromagnetic gauge field. We expect that the quantitative results discussed above would be slightly different upon including a gauge field, with the qualitative behavior remaining unchanged. Qualitatively, we expect that the electromagnetic interactions would enhance that stability of the vortons because the electromagnetic charge of the K^+ condensate trapped in the core gives an additional contribution $\sim Q^2/R$ and prevents the vortons from shrinking.

IV. DOMAIN WALLS, DRUM VORTONS, AND MAGNUS FORCES

In the previous section we have demonstrated that loops of superconducting K strings, called vortons, can exist as classically stable objects due to the fact that charges and currents are trapped on the string core. We will now include a brief discussion of other effects that are important in order to have a correct description of vortons in the CFL+ K^0 phase of high density QCD.

Up to this point, we have not included terms in the Lagrangian that explicitly break the $U(1)_Y$ symmetry. If the weak interactions are taken into account, there is a small piece which must be added to the effective Lagrangian (7) which explicitly breaks the $U(1)_Y$ symmetry [8]:

$$\delta\mathcal{L} = -V(\varphi) = f_\pi^2 m_{\text{dw}}^2 \cos\varphi, \quad (35)$$

where φ is the phase of K^0 and m_{dw} will be given below. As described in full detail in [8], this leads to the formation of domain walls, with the phase φ varying from 0 to 2π across the wall (the same vacuum state exists on both sides of the wall). Consequently, this leads a domain wall being attached to every string. Therefore, as one encircles the string at large distances from the core the phase variation from 0 to 2π is not uniform but is sandwiched inside a domain wall of width m_{dw}^{-1} , with m_{dw}^{-1} set by the coefficient of the explicit symmetry breaking term. We will simply state the results of Son [8] here without going into details. The domain wall tension α_{dw} (energy per unit area) is given by:

$$\begin{aligned} \alpha_{\text{dw}} &= 8vf_\pi^2 m_{\text{dw}}, \\ m_{\text{dw}}^2 &= \frac{162\sqrt{2}\pi}{21-8\ln 2} \frac{G_F}{\alpha_s} \cos\theta_c \sin\theta_c m_u m_s \Delta^2, \end{aligned} \quad (36)$$

where G_F is the Fermi constant, θ_c is the Cabibbo angle, and α_s is the strong coupling constant. The inverse mass m_{dw}^{-1} is approximately the width of the domain wall. Son calculates $m_{\text{dw}} \sim 50$ keV for physical values of the relevant parameters [8]. If the size of the domain wall is greater than the thickness, $R \gg m_{\text{dw}}^{-1}$ for a circular domain wall of radius R , then the total energy of this configuration can be approximated as:

$$E_{\text{dw}} \simeq \pi R^2 \alpha_{\text{dw}} \quad (37)$$

Since every string must be attached to a domain wall, the vortons discussed in the previous section will have a domain wall stretched across their surface like a soap bubble. Similar configurations have been recently studied in the linear sigma model at nonzero temperature [28] and have been referred to as “drum vortons”. In the case that there is no domain wall attached to the vorton, the minimization of the energy with respect to R leads to the result that $k = N/R$ is const for the chiral case $vk = \omega$, independent of R . Therefore, the vorton could have an arbitrarily large size. The presence of the domain

wall will lead to an upper bound on the radius of these vortons. Now that we have an approximate expression to the domain wall contribution to the energy, we can add Eqs. (30), (31), and (37) to arrive at an expression for the total energy of a circular drum vorton of radius R which is valid for $R \gg m_{\text{dw}}^{-1}$:

$$\begin{aligned} E &= E_{K^0} + E_{K^+} + E_{\text{dw}} \\ &= 2\pi^2 R \eta^2 v^2 \ln(\beta R) + \pi R^2 \alpha_{\text{dw}} \\ &\quad + \frac{1}{R} \left(\frac{Q^2}{2\pi\Sigma} + 2\pi v^2 N^2 \Sigma \right). \end{aligned} \quad (38)$$

This expression must be minimized with respect to R to find the size of these classically stable objects. This problem is quite complicated because of a number simplifications we have made in eq. (38). In particular, eq. (37) is not literally valid for relatively small $R_0 \sim m_{\text{dw}}^{-1}$ when equilibrium is reached (see below).

We will now estimate the typical size of a vorton. We start with relatively small charges Q, N (and correspondingly R) when the domain wall contribution can be neglected, and the equilibrium is reached at R_0 given by eq. (32). We slowly increase Q and N such that domain wall contribution becomes of the same order of magnitude as the string-related terms. This happens when $Q, N \sim f_\pi/m_{\text{dw}} \gg 1$. The size of the configuration at this point $R_0 \sim Q/f_\pi \sim m_{\text{dw}}^{-1}$ reaches the magnitude of the domain wall width, i.e. $(50 \text{ keV})^{-1}$ which is much larger than any QCD-related scale of the problem. If one increases Q and N further, the first term in eq.(38) becomes irrelevant, and equilibrium is achieved when $R_0^3 \sim Q^2/(\Sigma\alpha_{\text{dw}})$ at which point the energy of configuration $E \sim Q^{4/3}\alpha^{1/3}$ grows too fast with Q . Such a large configuration will decay to smaller vortons by preserving the charges Q and N , decreasing the total energy. Therefore, one expects that the maximum vorton size is related to the weak interactions which set the typical vorton scale to be m_{dw}^{-1} .

There exists an additional force which may further stabilize the vortons. This is the Magnus force, which arises when a global string moves through a Lorentz-noninvariant fluid. We naturally have such a background, since we are working at nonzero chemical potential, which breaks Lorentz invariance. The corresponding expression has been derived in ref. [38] where it was demonstrated that in the language of the Goldstone boson such a background corresponds to a time dependent phase of the order parameter. This phase in our notations takes the form $\sim e^{i\mu_{\text{eff}}t}$. If vorton moves with velocity \vec{v} through this fluid, the force exerted on the vorton per unit length:

$$\vec{F} = 2\pi\gamma\eta^2\mu_{\text{eff}} \vec{v} \times \vec{m}, \quad (39)$$

where γ is the standard relativistic factor and \vec{m} is the circulation vector of unit magnitude, $|\vec{m}| = 1$, which points in the direction of the string. If the velocity vector \vec{v} is perpendicular to the plane formed by the vorton, then

there will be a Magnus force present which points outward, further stabilizing the vorton. This will in turn increase the size of the vorton. If the vorton moves in the opposite direction, the Magnus force points inward, which decreases the size of the vorton.

Finally, the issue of quantum stability of vortons has not been addressed in this paper. In the pure current case ($Q = 0, N \neq 0$), the instanton solution has been explicitly constructed and the lifetime calculated analytically [35]. The decay mechanism is a quantum mechanical tunneling process where the condensate goes to zero on the core, allowing the winding number to decrease from N to $N - 1$. However, the vortons discussed here have nonzero Q, N so the results obtained in [35] do not apply. In spite of this fact, we expect that the vortons in high density QCD discussed in this paper are long lived due to the approximate “chiral” relation (27) which must be satisfied in order to have superconducting strings. However, at the moment we cannot make any definitive statements on the lifetime of the vortons discussed in this paper. In order to make such estimations we need to understand the vorton interactions, which were completely ignored in this work. To understand the dynamics of vortons we need to know: first of all, the interaction of the Goldstone particles with the vorton. This would allow us to calculate the corresponding cross section which is important for the analysis of the frictional force acting on a moving vorton. Secondly, the same interaction would allow us to estimate the Goldstone mode production by the vorton. This knowledge is essential for the study of the Goldstone boson radiation from moving vortons. Finally, the interaction is essential for studying such issues as the typical life time of a vorton, the typical behavior of vortons when they can join/disjoin with each other and absorb/emit the Goldstone bosons. The quantum numbers (N, Q, M) should be conserved in all the processes mentioned above. Unfortunately, none of these questions can be answered at this point.

V. CONCLUSION

In this paper we have shown that loops of superconducting K strings [10, 12], called vortons, can exist as classically stable objects within the CFL+ K^0 phase of high density QCD. The main mechanism which stabilizes these superconducting K -string loops is the presence of charge and current which is trapped on the string. The main difference between these vortons and vortons within other models is the presence of a domain wall which is stretched across the surface. These domain walls set up upper bound on the allowable vorton size which is m_{dw}^{-1} in contrast with cosmic vortons which could become arbitrary large in size.

The most intriguing aspect of these vortons is their ability to carry angular momentum due to the presence of nonzero charge and current trapped in the core. Moreover, the vortons are very efficient carriers of the angular

momentum. Indeed, to simplify our estimates in what follows, we use Λ_{QCD} to be a typical scale of the problem, it could be any of dimensional parameters (or their combination) discussed above such as $\mu_{\text{eff}}, \Delta, k, \omega, f_\pi, \dots$ etc. As we demonstrated above, the angular momentum carried by a single vorton of size L is $M \sim \Lambda_{QCD}^2 L^2$ (see eq. (34)), and grows proportional to the area L^2 up to a maximal possible size, which is $\bar{L} \sim m_{\text{dw}}^{-1}$. As we mentioned, the parameter m_{dw} has a characteristic scale of the weak interactions, and it is three order of magnitudes smaller than Λ_{QCD} . At the same time, the energy of the vorton scales linearly with the size, $E \sim \Lambda_{QCD}^2 L$ as long as $L \leq \bar{L} \sim m_{\text{dw}}^{-1}$. Therefore, the angular momentum per energy scales as $M/E \sim L$ for $L \leq \bar{L} \sim m_{\text{dw}}^{-1}$. At the same time, a typical straight vortex (not a vorton) is expected to carry the angular momentum according to the relation $M \sim \Lambda_{QCD} L$ and $E \sim \Lambda_{QCD}^2 L$, such that $M/E \sim \Lambda_{QCD}^{-1}$. Therefore, the vortons are much more efficient carriers of the angular momentum than any regular straight vortices. In addition to this, the larger the vorton, the more efficient they become at carrying angular momentum. However, as explained above there is a maximum vorton size before they become unstable; it is $\bar{L} \sim m_{\text{dw}}^{-1}$.

Therefore, one should expect that most of the vortons in the core of a neutron star would have one and the same typical size, which is $\bar{L} \sim m_{\text{dw}}^{-1}$. As discussed above, the vortons ability to carry angular momentum efficiently makes them the important dynamical degrees of freedom. In particular, they might be the key elements for the explanation of phenomenon such as glitches. The same vortons might be important objects for other problems such as describing the dynamics of the electromagnetic fields in cores of the neutron stars (vortons are positively charged configurations due to a K^+ condensate trapped in the vortex). The vortons could be important for discussions of transport properties, as well as problems related to the cooling of the system. This is due to the fact that a vorton is a relatively large configuration with fields correlated over large distances (in QCD units). In such a case, as is known, the cross-section for the particle scattering by strings, could be very large, and could influence the cooling of the system. Finally, the vortons might be the only possible carriers of the angular momentum in the crystalline superconducting phase [39]. Indeed, in this phase it is quite difficult (if possible at all) to construct a regular straight vortex which can carry the angular momentum. To conclude: we believe that the physics of vortons could prove to be interesting for compact astrophysical objects such as neutron stars where CFL phase is likely realized. We hope that the present paper will initiate some activity in the direction of the phenomenological implications of vortons.

Furthermore, we believe that the study of such objects is important due to a completely different reason. Vortons were originally introduced in the context of cosmology [21, 25, 26] and more recently within the $SO(5)$

theory of high T_c superconductivity [30]. In this paper we argued that vortons may play an important role in astrophysics. The analogy between all these fields provides another example where astrophysical and cosmological phenomena have similarities with systems in condensed matter physics, and therefore, may be studied by doing laboratory experiments. For further details on recent condensed matter experiments designed in order to test ideas drawn from cosmology we refer the reader to the review papers [31, 32].

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